1 a. U : I will move to Utrecht

G : I will move to Groningen

P : You persuade me

Translation: $(\lnot U \land \lnot G) \lor P$

Also correct: $\lnot(U \lor G) \lor P$

$\lnot P \rightarrow \lnot(U \lor G)$

$\lnot P \rightarrow (\lnot U \land \lnot G)$

b. B : Ben Feringa will travel to Stockholm

J : Janine Jansen will play at the Nobel concert

Translation: $B \Leftrightarrow J$

2. Translation key: c : Carlsen

k : Karjakin

$B(x,y)$: x has beaten y

$G(x)$: x is a grandmaster

$M(x,y)$: x is better than y

$F(x,y)$: x is a Facebook friend of y

$L(x)$: x has followed the match

$P(x,y,z)$: x prefers y over z

a) $B(c,k) \rightarrow \exists x\,(G(x) \land \forall y\,((G(y) \land y \neq x) \rightarrow M(x,y)))$

b) $\forall x\,(F(x,c) \rightarrow L(x)) \land \lnot \forall x\,(F(x,c) \rightarrow P(x,c,k))$

It will also be counted correct if you use a constant: a : the match

$L(x,y)$: x has followed y

$$\forall x\,(F(x,c) \rightarrow L(x,a)) \land \underbrace{\lnot \forall x\,(F(x,c) \rightarrow P(x,c,k))}_{\Updownarrow \text{ also OK}}$$

,, ,, ,, $\land\ \exists x\,(F(x,c) \land \lnot P(x,c,k))$

Resit

3 a)

1 $(A \vee B) \to (C \wedge D)$

2 A

3 $A \vee B$ — $\vee$ Intro : 2

4 $C \wedge D$ — $\to$ Elim : 1, 3

5 C — $\wedge$ Elim : 4

6 $A \to C$ — $\to$ Intro : 2-5

7 B

8 $A \vee B$ — $\vee$ Intro : 7

9 $C \wedge D$ — $\to$ Elim : 1, 8

10 D — $\wedge$ Elim : 9

11. $B \to D$ — $\to$ Intro : 7-10

12. $(A \to C) \wedge (B \to D)$

b.

1. $R(a,b) \wedge R(b,a)$

2. $a = b$

3 $a = a$ — $=$ (Id.) Intro

4 $b = a$ — $=$ Elim: 3, 2

5. $R(a,a) \wedge R(a,a)$ — $=$ Elim: 1, 4

6 $R(a,a)$ — $\wedge$ Elim : 5

c.

1. $\forall x (P(x) \to Q(x))$

2. $\neg \exists z\, Q(z)$

3 $\exists y\, P(y)$

4 $\boxed{c}$, $P(c)$

5. $P(c) \to Q(c)$ — $\forall$ Elim : 1

6. $Q(c)$ — $\to$ Elim : 5, 4

7 $\exists z\, Q(z)$ — $\exists$ Intro : 6

8. $\perp$

9 $\perp$ — $\exists$ Elim : 3, 4-8

10. $\neg \exists y\, P(y)$ — $\neg$ Intro : 3-9

3d.

1	$\forall x\, R(x,x)$	
2	$\boxed{c}$	
3.	$R(c,c)$	$\forall$ Elim: 1
4	$\exists z\, R(z,c)$	$\exists$ Intro: 3
5.	$\forall w\, \exists z\, R(z,w)$	$\forall$ Intro: 2-4

4 a)

	A	B	C	$(A\land\neg B)$	$\leftrightarrow$	$(C\land\neg B)$	$(A\leftrightarrow C)$	$\land$	$\neg B$
1.	T	T	T	F F	T	F F	T	F	F
2.	T	T	F	F F	T	F F	F	F	F
3.	T	F	T	T T	T	T T	T	T	T
4.	T	F	F	T T	F	F T	F	F	T
5.	F	T	T	F F	T	F F	F	F	F
6.	F	T	F	F F	T	F F	T	F	F
7.	F	F	T	F T	F	T T	F	F	T
8.	F	F	F	F T	T	F T	T	T	T

The conclusion is not a tautological consequence of the premise, because there is at least one valuation (actually there are four) that makes $(A\land\neg B)\leftrightarrow(C\land\neg B)$ true but makes $(A\leftrightarrow C)\land\neg B$ false.
These situations are given as rows 1, 2, 5, 6 above.
(any one of them suffices as counterexample)

translation key: $H(x,y)$: x is husband of y
$succ(x)$: the Successor of x
b : George H.W. Bush
$f(x)$: the father of x

6a. $\neg \exists x \exists y \exists z (y = succ(z) \wedge H(x,y))$

It will also be correct if you use a function $h(x)$: the husband of x

$\neg \exists x \exists y \exists z (y = succ(z) \wedge x = h(y))$

b. $b = f(succ(succ(succ(b))))$

7 a. $\exists x (Cube(x) \wedge Small(x) \wedge \exists y \exists z (Dodec(y) \wedge Dodec(z) \wedge y \neq z \wedge SameRow(x,y) \wedge SameRow(x,z)))$

b. (i) F (iii) F
(ii) F (iv) T

c. The sentence is already true and remains true if you remove any object except c

8 a) $\forall x P(x) \leftrightarrow Q(c) \iff$
$(\forall x P(x) \wedge Q(c)) \vee (\neg \forall x P(x) \wedge \neg Q(c)) \iff$
$(\forall x P(x) \wedge Q(c)) \vee (\neg \forall z P(z) \wedge \neg Q(c)) \iff$
$\forall x (P(x) \wedge Q(c)) \vee (\exists z \neg P(z) \wedge \neg Q(c)) \iff$
$\forall x (P(x) \wedge Q(c)) \vee \exists z (\neg P(z) \wedge \neg Q(c)) \iff$
$\forall x [(P(x) \wedge Q(c)) \vee \exists z (\neg P(z) \wedge \neg Q(c))] \iff$
$\forall x \exists z [(P(x) \wedge Q(c)) \vee (\neg P(z) \wedge \neg Q(c))]$

This formula is in prenex normal form

b. For the formula $\exists x \forall y \forall z \exists u (R(x,y,z) \vee P(u))$, we introduce a constant c and a binary function g. Its Skolem form is: $\forall y \forall z (R(c, y, z) \vee P(g(y,z)))$

c. ①

A	B	C	D	E	$(\neg A \vee \neg B \vee \neg C) \wedge (B \vee \neg D) \wedge C \wedge (\neg E \vee A) \wedge (E \vee \neg C)$
T	F	T	F	T	F T F F T T F T T F
3d	4	2	4	3b	3d 4 3a 4 4 2 3c 3d 3b 3a

The formula is satisfiable

9 a) $\mathcal{M} \models P(x) \wedge R(x,z)\,[h[x/4]] \overset{\wedge\text{-clause}}{\iff}$

$\mathcal{M} \models P(x)\,[h[x/4]]$ and $\mathcal{M} \models R(x,z)\,[h[x/4]] \overset{\text{atomic}}{\iff}$

$[\![x]\!]^{\mathcal{M}}_{h[x/4]} \in \mathcal{M}(P)$ and $\langle [\![x]\!]^{\mathcal{M}}_{h[x/4]}, [\![z]\!]^{\mathcal{M}}_{h[x/4]} \rangle \in \mathcal{M}(R) \iff$

$4 \in \mathcal{M}(P)$ and $\langle 4,3\rangle \in \mathcal{M}(R)$.

This sentence is false because the second conjunct is false: $\langle 4,3\rangle \notin \mathcal{M}(R)$.

b) $\mathcal{M} \models \forall x \exists y \neg R(x,y)\,[h] \overset{\forall\text{-clause}}{\iff}$

For all $d \in \mathcal{M}(\forall)$, $\mathcal{M} \models \exists y \neg R(x,y)\,[h[x/d]] \overset{\exists\text{-clause}}{\iff}$

For all $d \in \mathcal{M}(\forall)$ there is an $e \in \mathcal{M}(\forall)$ s.t. $\mathcal{M} \models \neg R(x,y)\,[h[x/d, y/e]]$

$\overset{\neg\text{ clause}}{\iff}$ For all $d \in \mathcal{M}(\forall)$ there is an $e \in \mathcal{M}(\forall)$ such that not $\mathcal{M} \models R(x,y)\,[h[x/d, y/e]]$

$\overset{\text{atomic}}{\iff}$ For all $d \in \mathcal{M}(\forall)$ there is an $e \in \mathcal{M}(\forall)$ s.t. not $\langle [\![x]\!]^{\mathcal{M}}_{h[x/d, y/e]}, [\![y]\!]^{\mathcal{M}}_{h[x/d, y/e]} \rangle \in \mathcal{M}(R)$

$\iff$ For all $d \in \mathcal{M}(\forall)$ there is an $e \in \mathcal{M}(\forall)$ such that not $\langle d,e\rangle \in \mathcal{M}(R)$.

This sentence is true:

- For $d=1$, take $e=2$ (or $e=3$ or $e=4$), then $\langle d,e\rangle \notin \mathcal{M}(R)$
- For $d=2$, take $e=1$ (or $e=4$), then $\langle d,e\rangle \notin \mathcal{M}(R)$
- For $d=3$, take $e=1$ (or $e=2$ or $e=4$), then $\langle d,e\rangle \notin \mathcal{M}(R)$
- For $d=4$, take any $e \in \mathcal{M}(\forall)$, then $\langle d,e\rangle \notin \mathcal{M}(R)$

c) $\mathcal{M} \models \exists y \forall x (P(y) \to (R(y,x) \vee R(x,z)))\,[h] \overset{\exists\text{-clause}}{\iff}$

There is a $d \in \mathcal{M}(\forall)$ such that $\mathcal{M} \models \forall x (P(y) \to (R(y,x) \vee R(x,z)))\,[h[y/d]] \overset{\forall\text{-clause}}{\iff}$

There is a $d \in \mathcal{M}(\forall)$ such that for all $e \in \mathcal{M}(\forall)$,

$\mathcal{M} \models P(y) \to (R(y,x) \vee R(x,z))\,[h[y/d, x/e]] \overset{\to\text{ clause}}{\iff}$

There is a $d \in \mathcal{M}(\forall)$ such that for all $e \in \mathcal{M}(\forall)$, (*)

If $\mathcal{M} \models P(y)\,\cancel{h}[y/d, x/e]$, then $\mathcal{M} \models R(y,x) \vee R(x,z)\,[h[y/d, x/e]]$

$\iff$ (*) If $[\![y]\!]^{\mathcal{M}}_{h[y/d, x/e]} \in \mathcal{M}(P)$ then $(\mathcal{M} \models R(y,x)\,[h[y/d, x/e]]$ or $\mathcal{M} \models R(x,z)_{h[y/d, x/e]})$

$\iff$ (*) If $d \in \mathcal{M}(P)$, then $(\langle d,e\rangle \in \mathcal{M}(R)$ or $\langle e, h(z)\rangle \in \mathcal{M}(R))$

$\iff$ There is a $d \in \mathcal{M}(\forall)$ such that for all $e \in \mathcal{M}(\forall)$,

If $d \in \mathcal{M}(P)$ then $(\langle d,e\rangle \in \mathcal{M}(R)$ or $\langle e,3\rangle \in \mathcal{M}(R))$

This sentence is true. Just take $d=1$ or $d=2$, then $d \notin \mathcal{M}(P)$, so the conditional is true (whatever the truth value of the consequens)

10. Solution:

Chris – Law – 30 EC – 9:30
Alex – Computing Science (CS) – 25 EC – 10:00
David – Math – 5 EC – 10:30
Bobby – Astronomy – 15 EC – 11:00 (woman)

a) Alex had a beer with Chris and David - indeed 3 different students
b) Bobby, who indeed does not study Math but Astronomy, has 15 EC, which is precisely 3 × the 5 EC that David, the Math student has.
c) Bobby submitted at 11:00, so indeed not at 10:00
d) Alex has 25 EC, which is indeed more than David's 5 EC
e) The CS student Alex submitted at 10:00, indeed earlier than David who has 1/5 of Alex' EC (namely 5 EC is 1/5 × 25) & submitted at 10:30
f) David the math student handed in at 10:30 – not 10:00 – and Bobby the woman with 15 EC handed in at 11:00 – not 10:00.
g) Chris, who submitted at 9:30, has 30 EC, which is exactly twice as many as the 15 EC of Bobby, the Astronomy student.
h) Bobby, the student who submitted at 11:00 is not a man and does not study Law but Astronomy
i) David the Math Student handed in at 10:30, so he was not the last (that was Bobby at 11:00)